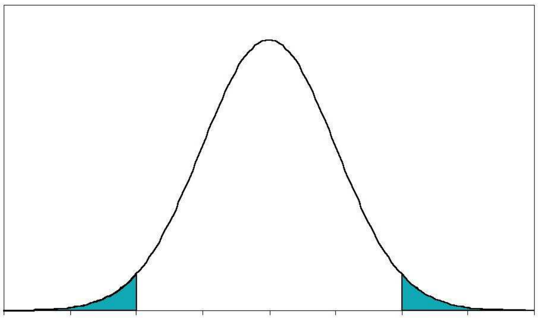
15 –Hypothesis Testing, Part II

For population proportion and population standard deviation hypothesis testing, the protocol is largely the same as it was for population mean, and there is no choice of distributions to use. The differences are in the details of each step. Here are those differences:

- For population proportion, the *z* distribution is used in all cases in the formula $z = \frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$.
- For population standard deviation, the χ^2 distribution is used in all cases in the formula $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$.

Example 1: Of the US households with televisions, Nielsen Media suggests that 83% of them have two or more. However, a local cable company found that 240 of 300 local customers had two or more televisions. Perform a hypothesis test with 95% confidence to determine if 0.83 is accurate.



Solution:

Step 1: H_0: $p = 0.83$; H_1: $p \neq 0.83$. This is a two tailed test.

Step 2: Because the confidence level is 95%, we need to find the *z*-score that creates an area of 0.025 in each tail. The *z* table gives the values $z = -1.96$ and $z = 1.96$.

Step 3: The test value is $z = \frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ with the parameters $p = 0.83$, $\hat{p} = \frac{240}{300} = 0.80$, and $n = 300$. That computation is $z = \frac{0.80 - 0.83}{\sqrt{0.83(0.17)/300}} \approx -1.38$

Step 4: Since -1.38 is **not** in either tail, we do **not** reject the null hypothesis. Our conclusion is that we have do not have sufficient evidence to indicate that the proportion is different from 0.83.

For argument's sake, the *p*-value for the above problem is found by looking -1.38 up on the *z* chart. The value identified, 0.0838, is the area to the left of -1.38. Since this is a two-tailed test, that number must be doubled, and so $p = 0.1676$. H_0 would be rejected at a confidence level of 80%, but not for much higher than that.

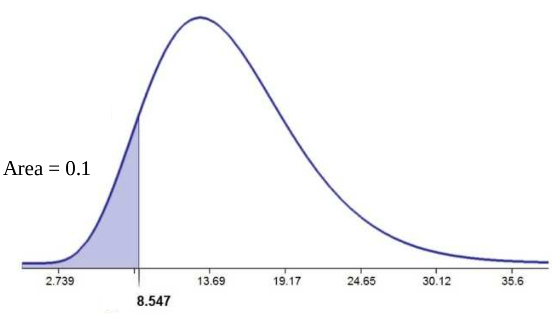
Example 2: A telecommunications advisor claims that the standard deviation of call transfer times for a company's phones is 15 seconds (0.25 minutes). In a sample of 16 calls, the standard deviation of the call transfer times is 12 seconds (0.2 minutes). At a significance level of 0.10, is the standard deviation actually shorter than the advisor claims?

Solution:

Step 1: H_0: $\sigma = 0.25$; H_1: $\sigma < 0.25$. This is a left-tailed test.

Step 2: We need to use the χ^2 distribution. Using $df = 15$ to find the χ^2 value with 0.90 to the right, the boundary of the left tail is 8.547.

Step 3: The test value is computed with the formula $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$. The parameters are $n = 16$, $s = 0.2$, $\sigma = 0.25$ and so $\chi^2 = \frac{(15)\times 0.2^2}{0.25^2} \approx 9.60$.



Step 4: Since this value is **not** in the left tail we do **not** reject H_0. We do not have sufficient evidence to suggest with 90% confidence that the standard deviation of call transfer times is shorter than 15 seconds.

Further $p > .10$ since $9.60 > 8.547$ in the $df = 15$ row of the χ^2 distribution table. In other words, there is no reasonable confidence level for which we would reject H_0.