

Put any justification or explanations on the back, or separate sheets.

- (1) Prove that $0.9999999 \dots$ is exactly equal to 1.

- (2) Prove that $0.4999999 \dots$ is exactly equal to $\frac{1}{2}$.

- (3) Let E be the set of Even Natural Numbers.
 - a. Is E closed under addition? Show why or why not.

 - b. Is E closed under multiplication? Show why or why not.

 - c. Is E closed under subtraction? Show why or why not.

- (4) Consider the “max” operation on the Natural Numbers defined by $m(a, b)$, which simply returns the maximum of a and b . For example, $m(4, 3) = 4$, $m(1, 2) = 2$, and $m(6, 5) = 6$.
 - a. Is m commutative? Explain why or why not.

 - b. Is there an identity element for m ? If so, what is it? If not, explain why.

(5) Consider the operation R on the Set of Natural Numbers, with zero included, defined as follows:

aRb is equal to the remainder when b is divided into a

(for example, $5R1 = 0$, $5R2 = 1$, and $8R5 = 3$)

- a. There is an identity element for R. What is it? Show how you know.

- b. For any Natural Number n , what is the inverse of n ? Is this inverse unique?

- c. Is R distributive over addition? Show why or why not.

(6) Which property or properties are illustrated in each of the following expressions? Circle all that apply.

- | | | | | | | |
|-------------------------------|----|----|----|----|----|-----|
| • $2(3 + 7) = 2*3 + 2* 7$ | Cl | Co | As | Di | Id | Inv |
| • $3 - 5 = 5 - 3$ | Cl | Co | As | Di | Id | Inv |
| • $3 \times \frac{1}{3} = 1$ | Cl | Co | As | Di | Id | Inv |
| • $4 - 0 = 4$ | Cl | Co | As | Di | Id | Inv |
| • $\frac{2}{5}$ is rational | Cl | Co | As | Di | Id | Inv |
| • $4 - (5 + 3) = (4 - 5) - 3$ | Cl | Co | As | Di | Id | Inv |

(Cl = Closure, Co = Commutativity, As = Associativity, Di = Distributive, Id = Identity, Inv = Inverse)

(7) For the sequences below,

- Find the next two terms.
- Circle A, G, or F for Arithmetic, Geometric, or Fibonacci, or N for None of these.
 - For Arithmetic sequences, identify the common difference (d).
 - For Geometric sequence, identify the common ratio (r).
- Fill in the blanks

(1) $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$ A ($d = \underline{\hspace{1cm}}$) G ($r = \underline{\hspace{1cm}}$) F N

$a_8 = \underline{\hspace{1cm}}$ $a_n = \underline{\hspace{1cm}}$ $a_{62} = \underline{\hspace{1cm}}$

(2) $-2, 4, -8, 16, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$ A ($d = \underline{\hspace{1cm}}$) G ($r = \underline{\hspace{1cm}}$) F N

$a_9 = \underline{\hspace{1cm}}$ $a_n = \underline{\hspace{1cm}}$ $a_{28} = \underline{\hspace{1cm}}$

(3) $2\sqrt{3}, 6, 6\sqrt{3}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$ A ($d = \underline{\hspace{1cm}}$) G ($r = \underline{\hspace{1cm}}$) F N

$a_7 = \underline{\hspace{1cm}}$ $a_n = \underline{\hspace{1cm}}$ $a_{16} = \underline{\hspace{1cm}}$

(4) $\frac{2}{3}, 1, \frac{3}{2}, \frac{9}{4}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$ A ($d = \underline{\hspace{1cm}}$) G ($r = \underline{\hspace{1cm}}$) F N

$a_5 = \underline{\hspace{1cm}}$ $a_n = \underline{\hspace{1cm}}$ $a_{51} = \underline{\hspace{1cm}}$

(5) $0, \frac{2}{5}, \frac{2}{3}, \frac{6}{7}, 1, \frac{10}{9}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$ A ($d = \underline{\hspace{1cm}}$) G ($r = \underline{\hspace{1cm}}$) F N

$a_5 = \underline{\hspace{1cm}}$ $a_n = \underline{\hspace{1cm}}$ $a_{24} = \underline{\hspace{1cm}}$