

(1) Use the second derivative test to find the maximum and minimum values on the functions below:

a. $f(x) = x^3 - 12x$ on $[0, 4]$

b. $f(x) = \begin{cases} 2 - x^2, & 1 \leq x \leq 3 \\ 2 - 3x, & 3 < x \leq 5 \end{cases}$ on $[1, 5]$

c. $f(x) = 2x^3 + 2x + 1$ on $[-10, 10]$

d. $f(x) = \frac{1}{1+x^2}$ on $[-4, 4]$

e. $f(x) = \frac{1}{1-x^2}$

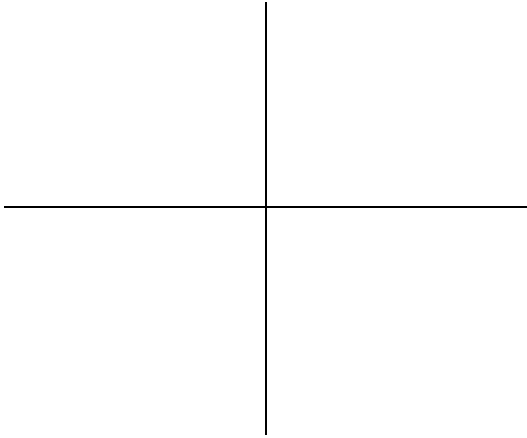
f. $f(x) = \frac{x^2+1}{x^2-1}$

g. $f(x) = 2x^3 + 2x + 1$ on $[-10, 10]$

(2) Sketch graphs of the four functions with ALL of the properties listed.

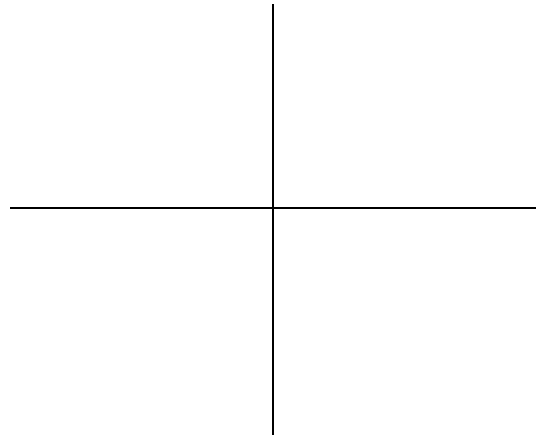
a.

- $f(2) = f(4) = 0$
- $f'(x) < 0$ for $x < 3$
- $f'(3)$ DNE
- $f'(x) > 0$ for $x > 3$
- $f''(x) < 0$ when $x \neq 3$



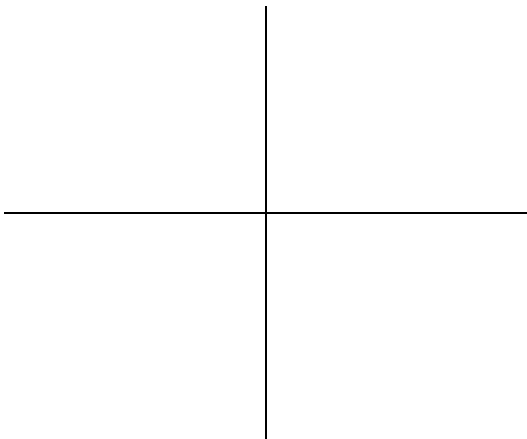
c.

- $f(0) = f(2) = 0$
- $f'(x) > 0$ for $x < 1$
- $f'(1) = 0$
- $f'(x) < 0$ for $x > 1$
- $f''(x) < 0$ everywhere



b.

- $f(2) = f(4) = 0$
- $f'(x) > 0$ for $x > 3$
- $f'(3)$ DNE
- $f'(x) < 0$ for $x < 3$
- $f''(x) > 0$ when $x \neq 3$



d.

- $f(0) = 0$
- $f'(0) = 0$ for $x = -5, 0, 5$
- $f(x)$ has vertical asymptotes at $x = \pm 3$
- $f'(x) < 0$ on $(-5, -3)$, $(-3, 3)$ and $(3, 5)$
- $f'(x) > 0$ on $(-\infty, -5)$ and $(5, \infty)$
- $f''(x) > 0$ on $(-3, 0)$ and $(3, \infty)$
- $f''(x) < 0$ on $(-\infty, -3)$ and $(0, 3)$

