

(1) Find the maximum and minimum values on the functions below:

a. $f(x) = x^3 - 12x$ on $[0, 4]$

b. $f(x) = \begin{cases} 2 - x^2, & 1 \leq x \leq 3 \\ 2 - 3x, & 3 < x \leq 5 \end{cases}$ on $[1, 5]$

(2) Use the first derivative test to find all maximum and minimum values of the functions below on the given intervals.

a. $f(x) = 2x^3 + 2x + 1$ on $[-10, 10]$

b. $f(x) = \frac{1}{1+x^2}$ on $[-4, 4]$

(3) Use the second derivative test to find all maximum and minimum values of the function $f(x) = \frac{1}{1-x^2}$ on the interval $(-1, 1)$.

(4) Use either the first or the second derivative test to find all maximum and minimum values of the functions below.

a. $f(x) = \frac{x^2+1}{x^2-1}$

b. $f(x) = 2x^3 + 2x + 1$ on $[-10, 10]$

(5) Sketch graphs of the four functions with ALL of the properties listed.

a.

- $f(2) = f(4) = 0$
- $f'(x) < 0$ for $x < 3$
- $f'(3)$ DNE
- $f'(x) > 0$ for $x > 3$
- $f''(x) < 0$ when $x \neq 3$

c.

- $f(0) = f(2) = 0$
- $f'(x) > 0$ for $x < 1$
- $f'(1) = 0$
- $f'(x) < 0$ for $x > 1$
- $f''(x) < 0$ everywhere

(6) Find two positive numbers such that the second is the reciprocal of the first and the sum is minimized.

(7) Give the details of each characteristic for the following functions

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|---|--|
| a. Domain of the function | f. Differentiability |
| b. x - and y -intercepts | g. Intervals of increase and decrease |
| c. Symmetry | h. Local maximums and local minimums |
| d. End behavior (limits at $\pm\infty$) including horizontal or slant asymptotes | i. Intervals of upward or downward concavity |
| e. Vertical Asymptotes | j. Inflection points |

- $f(x) = \frac{2(x^2-9)}{x^2-4}$

- $f(x) = \frac{x}{\sqrt{x^2+2}}$

- $f(x) = \frac{2x^3}{x^2+1}$

A box is to be made with a square base and no top. It must be made to hold 16 cubic inches of sand. Find the dimensions of the box of least surface area.

(8) Evaluate these indefinite integrals:

- $\int 3x^2 - 1 \, dx =$

- $\int (6x^4 + 2x^2)^4(6x^3 + x) \, dx =$

- $\int \frac{1}{\sqrt{x}} \, dx =$

- $\int \frac{x}{\sqrt{x^2+1}} \, dx =$

- $\int \frac{3}{x^3} \, dx =$

- $\int \frac{8x}{(5-3x^2)^2} \, dx =$

- $\int \frac{(2x)(5-3x^2)-(x^2+1)(-6x)}{(5-3x^2)^2} \, dx =$

- $\int (\sqrt{x} - \frac{1}{x})^3 (\frac{2}{\sqrt{x}} + \frac{4}{x^2}) \, dx =$

- $\int (x\sqrt{x})(\frac{-3}{x^4}) + (\frac{3\sqrt{x}}{2})(\frac{1}{x^3}) \, dx =$

- $\int \sqrt{\frac{x^{1/3+1}}{x^{4/3}}} \, dx =$

- $\int 5(6x^4 + 2x^2)^4(24x^3 + 4x) \, dx =$

- $\int \frac{1}{x^4 \left[\sqrt[3]{\left(\frac{3}{x^3}+3\right)^2} \right]} \, dx =$

(9) Find all solutions to these differential equations that are of the form $y = kx^n$.

- $y'' - \frac{y'}{x} = 0$

- $y'' + \frac{3y'}{x} = \frac{4}{x^2}$

- $y' = 2\sqrt{x}$

- $y'' = \frac{6y}{x^2}$