

Show all of your work in order to optimize credit.

**NO PHONES. NO CALCULATORS.**

Fractions, not decimals.

Limits

If  $\lim_{x \rightarrow a} f(x) = L$ , and  $\lim_{x \rightarrow a} g(x) = K$ , then ...

$$\lim_{x \rightarrow a} bf(x) = bL$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = L \pm K$$

$$\lim_{x \rightarrow a} f(x) \cdot g(x) = L \cdot K$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{L}{K} \text{ provided } K \neq 0$$

$$\lim_{x \rightarrow a} [f(x)]^n = L^n$$

Differentiation

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} [x^n] = nx^{n-1}$$

$$\frac{d}{dx} [c f(x)] = c f'(x)$$

$$\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\text{Product Rule: } \frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\text{Quotient Rule: } \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\text{Chain Rule: } \frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$$

(1) Compute the following limits:

a.  $\lim_{x \rightarrow 4} \frac{\frac{1}{2x} - \frac{1}{4}}{x-2} =$

b.  $\lim_{x \rightarrow 2} \frac{\frac{1}{2x} - \frac{1}{4}}{x-2} =$

c.  $\lim_{x \rightarrow 4} \frac{1}{(x-2)^2} =$

d.  $\lim_{x \rightarrow 2} \frac{1}{(x-2)^2} =$

- (2) Use the definition of the derivative (not the short cuts!) to confirm that  $\frac{d}{dx}[ax^2 + b] = 2ax$ .  
Show your work here:

- (3) Use the short-cut rules for differentiation to compute the derivatives of these functions:

a.  $\frac{d}{dx}\left(\frac{3}{\sqrt{x}} - x^4 + 5x\right) =$

b.  $\frac{d}{dx}\left(\frac{3x^4}{\sqrt{x+1}}\right) =$

c.  $\frac{d}{dx}\left[\sqrt[3]{x}\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x}}\right)\right] =$

- (4) The equation of the line tangent to the graph of the equation  $4xy - y^2 = 3$  at the point  $(1, 3)$  is

$$y = \underline{\hspace{2cm}} x + \underline{\hspace{2cm}}.$$

- (5) Two self-driving cars are driving *away from* an intersection in perpendicular directions. The first car is traveling north with velocity 5 meters per second. The second car is traveling east with velocity 8 meters per second. At the moment in time when the northbound car is precisely 15 meters from the intersection, how fast is the distance between the cars increasing (in meters per second)?

