6 – Probability, Part I

Probability (aka, likelihood) is the quantified chance of an event occurring.

Example 1: the chance of rain on any given day is expressed as a percentage, like 70%.

A probability experiment is that chance process. The outcome is the result of a probability experiment.

Example 2: Flipping a coin is a probability experiment, and the possible outcomes are *heads* and *tails*.

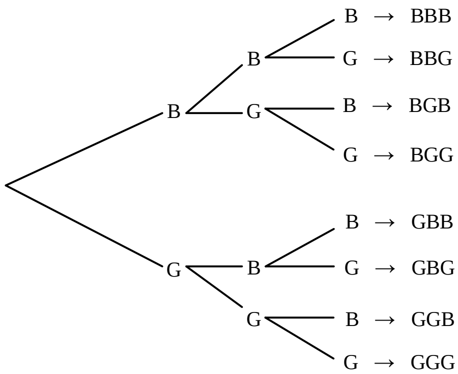
Each probability experiment has a sample space, denoted *S*, which is the set of all possible outcomes of that experiment. It will be important for later uses to ensure that every item in *S* has equal likelihood.

Example 3: Flip two coins. The sample space is *S* = {HH, HT, TH, TT}

Example 4: A family has three children.

The sample space is *S* = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

It is often useful to draw a tree diagram to determine the items in *S*. Here is a tree diagram for Example 4:



When discussing probability experiments, we are typically interested in some specific outcomes, or a subset of the sample space. Any subset of the sample space is called the event space, denoted *E*, and those items correspond to an event.

Example 4: In the probability experiment where a family has three children, the family may be most interested in the event defined by 2 boys and a girl. In this case, *E* = {BBG, BGB, GBB}.

Classical Probability: The probability that event *E* will happen is computed with the formula $p(E) = \frac{n(E)}{n(S)}$, where *n(E)* is the number of items in *E* and *n(S)* is the number of items in *S*.

Example 4: By counting the number of elements in E and S, we compute the probability that a family with three children has 2 boys and a girl to be $\frac{3}{8}$. Likewise, the probability that all 3 children are the same gender is $\frac{2}{8} = \frac{1}{4}$

Because of the way probability is computed, it has several properties:

- Probabilities will always be inclusively between 0 and 1 (i.e., $0 \leq p(E) \leq 1$)
- $p(E) = 0$ means that E will not happen
- $p(E) = 1$ means that E will definitely happen
- The sum of the probabilities of the items in *S* is 1

If *E* is an event within a probability experiment, then $\bar{E}$, pronounced *E*-bar, is the complement of E, or the subset of *S* that contains all items not in *E*. It represents the event that *E* does not happen. Furthermore, the probability of $\bar{E}$ is easily computed using this equation: $p(\bar{E}) = 1 - p(E)$. For example, if the probability of rain tomorrow is 70%, then the probability that it will not rain is 30%.

Another way to compute probability is to collect data. This is called empirical probability. If a probability experiment is done over and over again, data can to quantify the portion of outcomes that fit in the event space. This portion is the empirical probability.

Example 5: A coin is tossed 1000 times and lands heads up 462 of those times. The empirical probability of a coin landing heads is thus $\frac{462}{1000}$ = 0.462, or 46.2%. This should be contrasted with the classical probability of this experiment, which would be computed to be $\frac{1}{2}$ = 0.5, or 50%.

Within a probability experiment, we can discuss separate events, *A* and *B*. Two events are mutually exclusive because there is no overlap (i.e., no items in the sample space are in both *A* and *B*).

Example 4, let *A* be the event that all three children are of the same gender, and let *B* be the event that there are two boys and a girl. Also, let *C* be the event that the family has at least one boy. From these descriptions, we have the following event spaces:

A = {BBB, GGG}   B = {BBG, BGB, GBB}   C = {BBB, BBG, BGB, BGG, GBB, GBG, GGB}

And by counting these different event spaces, we get the following classical probabilities:

$p(A) = \frac{2}{8} = \frac{1}{4}$ $p(B) = \frac{3}{8}$ $p(C) = \frac{7}{8}$

When A and B are subspaces of the same sample space, we can compute the probabilities of A or B happening. That computation is: *p(A* or *B) = p(A) + p(B) – p(A* and *B).* And, if *A* and *B* are mutually exclusive, then *p(A* and *B)* = 0

Example 4*: p(A* or *B)* = $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$ since *p(A* and *B)* = 0. And, if you check you will see that 5 of the 8 items in *S* have either the same gender or two boys and a girl.

p(B or *C)* = $\frac{3}{8} + \frac{7}{8} - \frac{3}{8} = \frac{7}{8}$ since *p(B* and *C)* = $\frac{3}{8}$.

In fact, in this case, *P(B* or *C) = p(C)* and *P(B* and *C) = p(B).*