8 – Probability, Part III

The classical probability formula, $p(E) = \frac{n(E)}{n(S)}$, and the formulas for the *Fundamental Principle of Counting* (Permutations: ${}_nP_r = \frac{n!}{(n-r)!}$ and Combinations: ${}_nC_r = \frac{n!}{(n-r)!r!}$) can be used for computing probabilities.

The following examples show how:

Example 1: The numbers available for a padlock are each between 0 and 29. What is the probability that a 3-number combination for a padlock is 25 – 7 – 16?

Solution: There is only one correct sequence of numbers (25 – 7 – 16), so the numerator for probability is 1.The denominator is the number of ways to order the options for the padlock. We have 30 options for each number, and must choose 3. Since order matters, the denominator is 30×29×28, or ${}_{30}P_3 = \frac{30!}{27!}$. = 24360. The probability is $p = \frac{1}{24360} \approx .000041$.

Example 2: There are four male and three female executives at a company to be selected for a committee of three. What is the probability that the committee selected is all female? All male?

Solution: Since there are only three females, there is only one way to select all three of them. The number of ways to choose any 3 people from among the 7 executives is ${}_7C_3 = 35$.Thus, the probability is $\frac{1}{35} \approx .028$.

The number of ways to choose 3 men from the 4 male executives is ${}_4C_3 = 4$. So, the probability is $\frac{4}{35} \approx .114$

Example 3 (adapted from textbook problem #17, section 4.5): In order for holly plants to yield berries, a male and female must be planted near each other. A store has 12 holly plants for sale but the gender of each if the plants is not known. However, it is known that 8 of the twelve plants are female plants, and 4 are male plants. What is the probability that a customer buys 2 plants and the plants yield berries? What is the probability if they buy 3? If they buy 4?

Solution: We want the probability that both plants are not the same gender. So, if *A* represents the event that both plants are of the same gender, we are asked to compute $p(\bar{A})$, and $p(\bar{A}) = 1 - p(A)$.

In order to compute *p(A),* we must count the number of ways to select 2 plants at random and for them to be of the same gender. Really, *p(A) = p(MM or FF) = p(MM) + p(FF),* since these are mutually exclusive events.

In total there are 12 plants, so the number of ways to select any two plants is ${}_{12}C_2$.

There are 8 female plants, so the number of ways to choose 2 female plants from the 8 is ${}_8C_2$.

There are 4 male plants, so the number of ways to choose 2 male plants from the 4 is ${}_4C_2$.

Therefore, $p(FF) = \frac{{}_8C_2}{{}_{12}C_2}$ and $p(MM) = \frac{{}_4C_2}{{}_{12}C_2}$. We now have $p(A) = \frac{{}_8C_2 + {}_4C_2}{{}_{12}C_2}$

The solution is $p(\bar{A}) = 1 - p(A) = 1 - \frac{{}_8C_2 + {}_4C_2}{{}_{12}C_2} \approx 0.485$

The probability that three are selected and the plants yield berries is similarly $1 - \frac{{}_8C_3 + {}_4C_3}{{}_{12}C_3} \approx 0.727$

The probability that four are selected and the plants yield berries is similarly $1 - \frac{{}_8C_4 + {}_4C_4}{{}_{12}C_4} \approx 0.857$

Example 4: In the standard game of Poker, "two pair" is a 5-card hand with two pairs of matching cards, and a fifth card. What is the probability that 5 cards dealt from a standard deck contain two pairs?

Solution: First identify the denominator. This is the total number of 5-card sequences from a deck of 52, or ${}_{52}C_5$, since the order of the cards doesn't matter. To get the numerator, we must break the five cards into a sequence of tasks in order to utilize the FPC.

<u>Task 1</u>: Choose the matching first pair. <u>Task 2</u>: Choose the suits of the first matching pair. <u>Task 3</u>: Choose the matching second pair. <u>Task 4</u>: Choose the suits of the second matching pair. <u>Task 5</u>: Choose the fifth card.

The first matching pair will be either 2s, 3s, 4s, . . . , As. So, there are 13 possible matching pairs. The suit of the pair can come in any of ${}_4C_2$ ways (there are four suits, and two cards). The second pair will be one of any of the 12 remaining values (the pairs can't match each other, or else it would be four-of-a-kind, a different hand in Poker). The second matching pair can have suits arranged in any of ${}_4C_2$ ways. However, because the 1st and 3rd tasks were the same, the order of them must be considered. There will be two representations of each pair of pairs (for example, 5♠5♣6♥6♣ and 6♥6♣5♠5♣), so we must divide by two.

Lastly, the 5th card can be any of the remaining 44 cards that do not match any of the other four cards (otherwise there would be three of a kind in the hand).

The total number of two pair hands is $13\times{}_4C_2\times12\times{}_4C_2\div2\times44 = 123{,}552$

The probability of two pairs is $\frac{123552}{{}_{52}C_5} \approx 0.0475$.