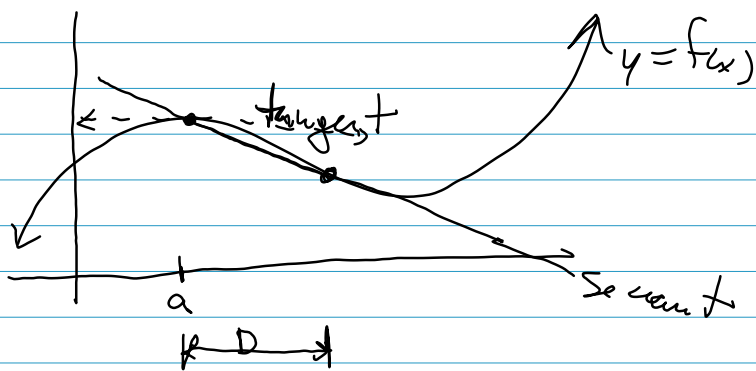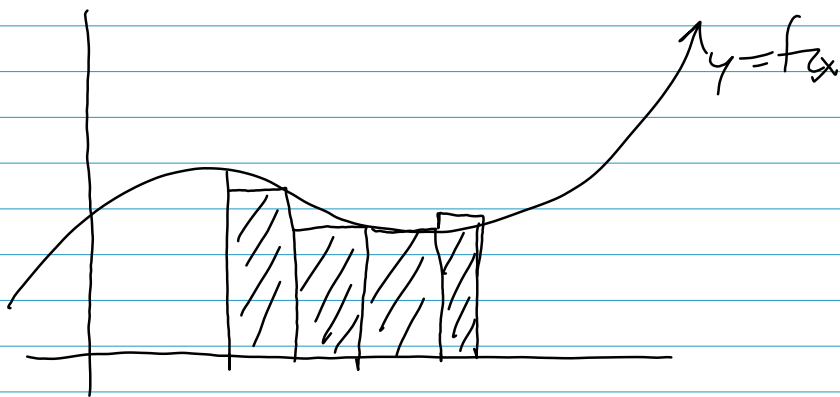Thursday, Jan 12th,

2 problems in Calculus

① Find the slope of a tangent line
to a curve.

we need __limits__
(distance D will
be zero)

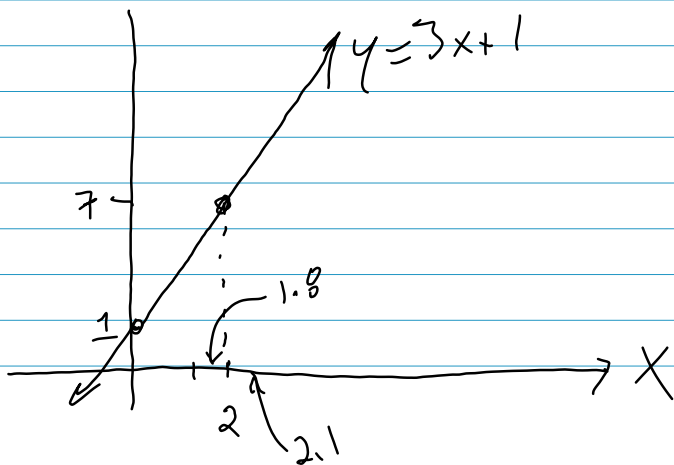(2) Find the area under a curve between 2 points.

I need ∞ rectangles.

$$\lim_{x \to a} f(x)$$

"the limit of f(x) as x approaches a
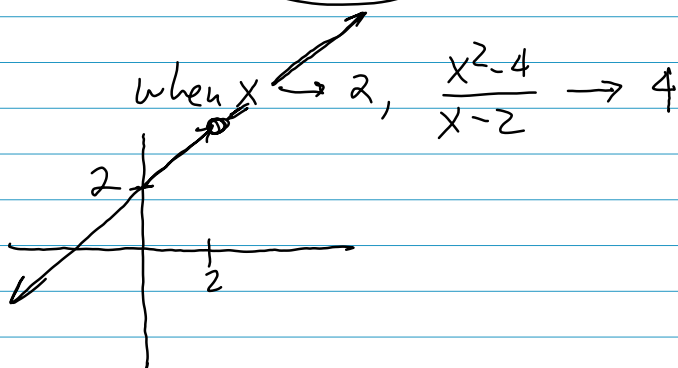
(the behaviour of f(x))

$$\lim_{x \to 2} 3x + 1 = 7$$
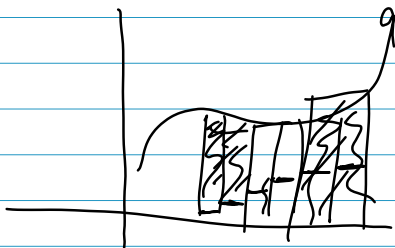
$$\lim_{x \to 2} \frac{x^2-4}{x-2} =$$

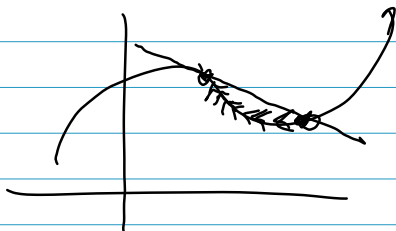$\uparrow$ $\frac{0}{0}$ is "indeterminate"
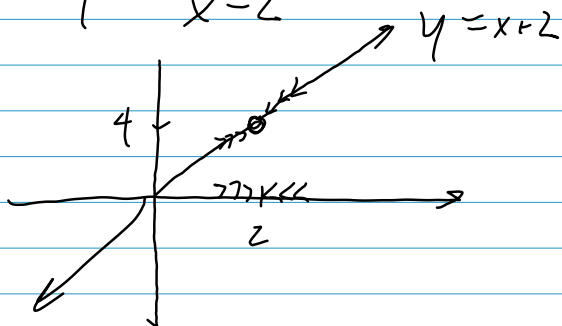
what does the graph of $\frac{x^2-4}{x-2}$

look like?

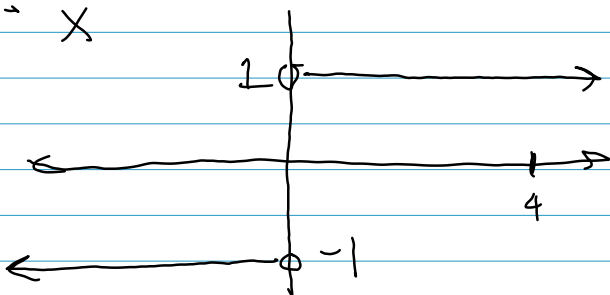$$\frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2} = \boxed{x+2}$$
$$(x \neq 2)$$

when $x \to 2$, $\frac{x^2-4}{x-2} \to 4$

$$y = \frac{x^2 - 4}{x - 2}$$

$y = x + 2$

4

2

$$f(x) = \frac{|x|}{x}$$

$$\lim_{x \to 4} f(x) = 1 \qquad \lim_{x \to -2} f(x) = -1$$

$$\lim_{x \to 3} f(x) = 1 \qquad \lim_{x \to -3} f(x) = -1$$

$$\lim_{x \to 0} f(x) \quad DNE$$

$$\lim_{x \to 0^+} f(x) = 1 \qquad \lim_{x \to 0^-} f(x) = -1$$

(right-sided limit)

$y = f(x)$

2

-1

3

-2

-1

$f(3) = 2$
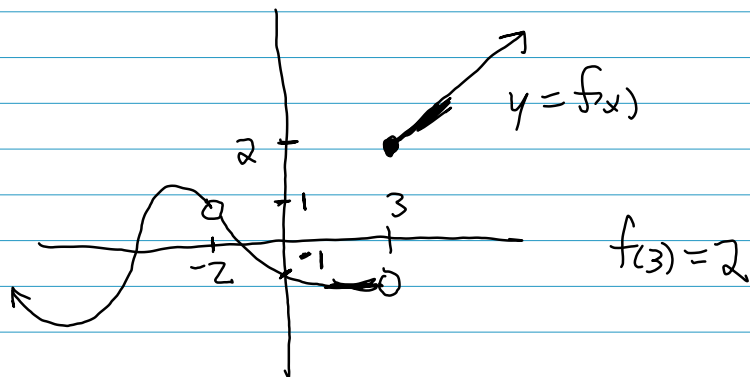
$$\lim_{x \to -7} f(x) = 1 \qquad \lim_{x \to 3} f(x) \ DNE$$
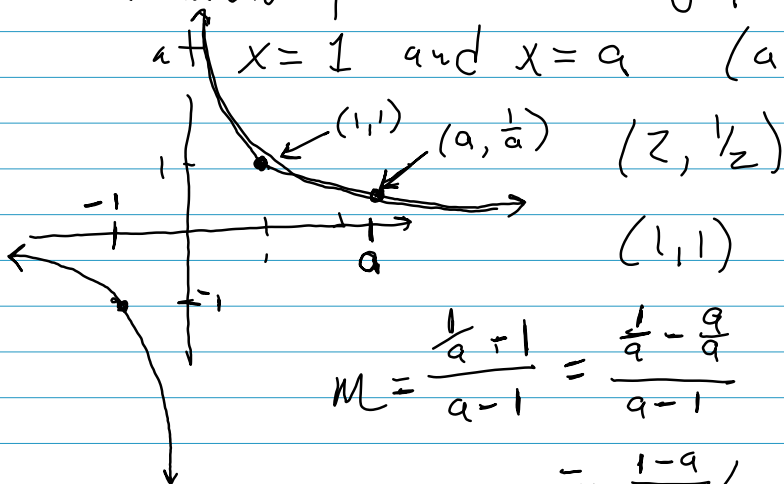
$$\lim_{x \to 0} f(x) = -1 \qquad f(0) = -1$$

$$\lim_{x \to 3^+} f(x) = 2 \qquad \lim_{x \to 3^-} f(x) = -1.2$$

$$\boxed{f(x) = \frac{1}{x}}$$

Find the slope of the line that
connects points on this graph
at $x = 1$ and $x = a$ $\quad (a > 1)$

$(1,1)$

$(a, \frac{1}{a})$

$(2, \frac{1}{2})$

$(1,1)$

$$M = \frac{\frac{1}{a} - 1}{a - 1} = \frac{\frac{1}{a} - \frac{a}{a}}{a - 1}$$

$$= \frac{1-a}{a} \Big/ a - 1$$

$$= \frac{1-a}{a} \times \frac{1}{a-1}$$

$$= \frac{-(a-1)}{a(a-1)} = -\frac{1}{a}$$