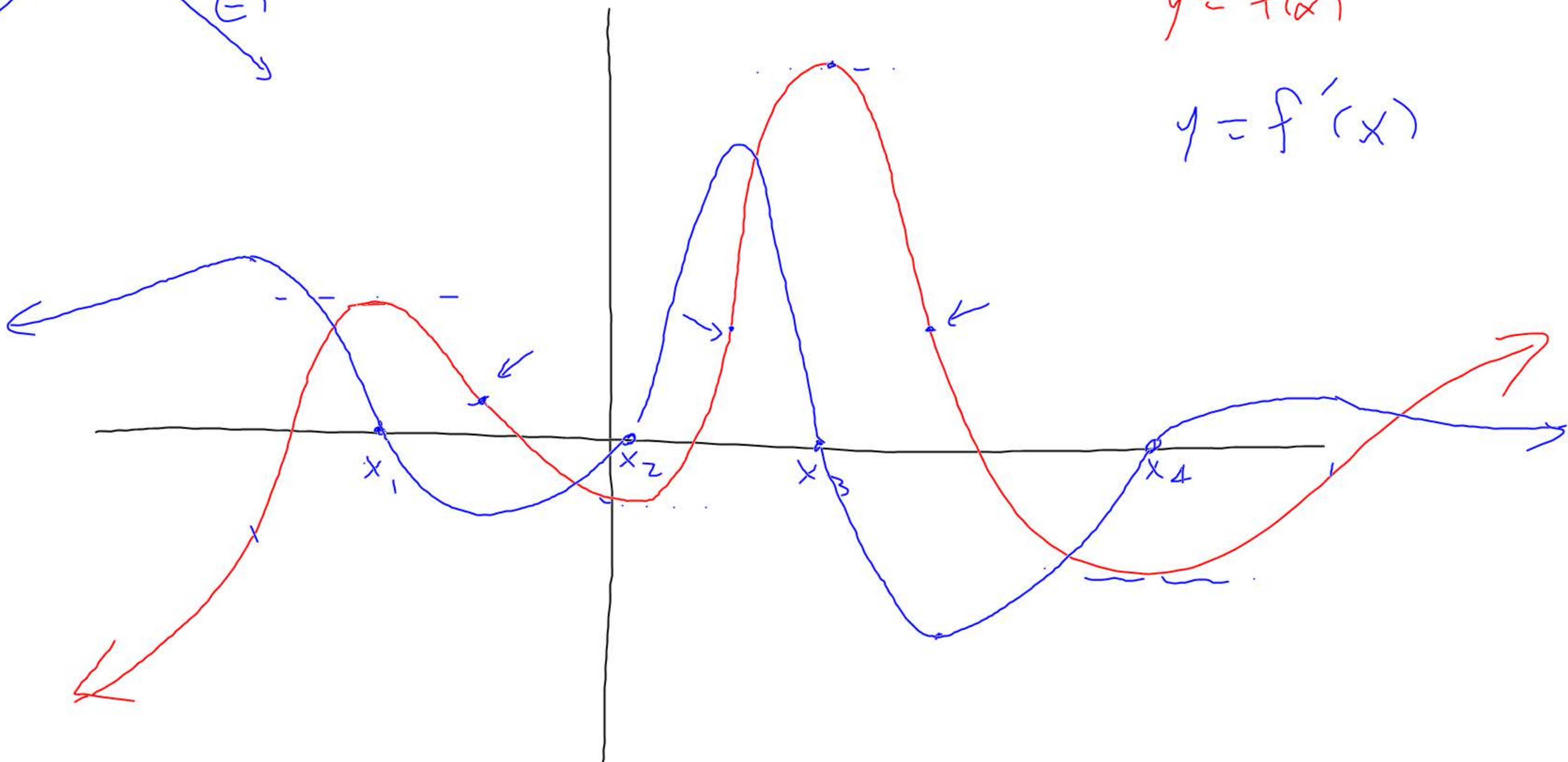


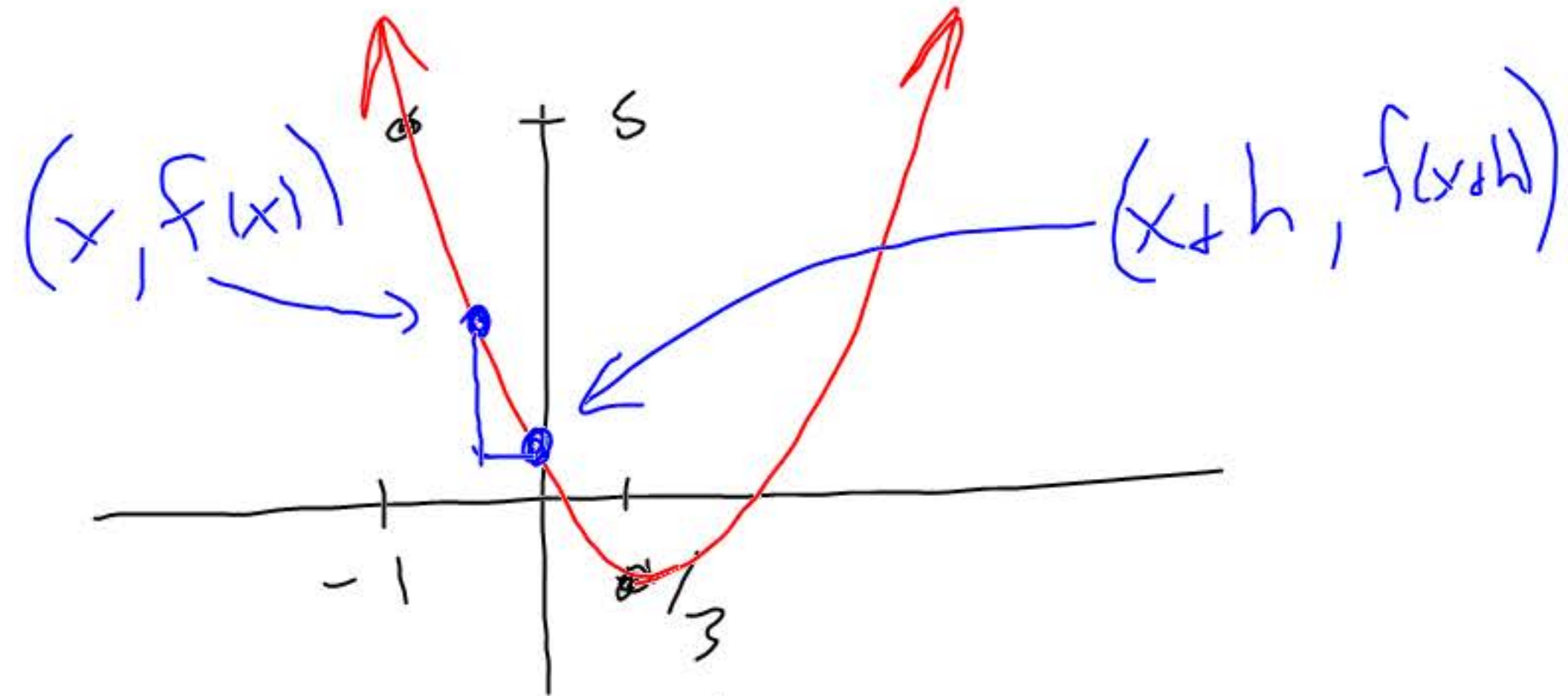
(+) ↗
(-) ↘

$$y = f(x)$$

$$y = f'(x)$$



$$f(x) = 3x^2 - 2x$$



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{x+h - x}$$

$$= \lim_{h \rightarrow 0} \frac{[3(x+h)^2 - 2(x+h)] - [3x^2 - 2x]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h - 3x^2 + 2x}{h}$$

$$f\left(\frac{1}{3}\right) = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 2h}{h} = \lim_{h \rightarrow 0} \frac{h(6x + 3h - 2)}{h} = 6x - 2$$

Notation → prime: the derivative

$$f'(x) = \frac{d}{dx} [f(x)]$$

(Newton)

← the derivative of f
(Leibniz)

$$\frac{d}{dx} [3x^2 - 2x] = 6x - 2$$

$$\frac{d}{dx} [\sqrt{x}] = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x+h} - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} [x^n] = n x^{n-1}$$

$$\frac{d}{dx} [x^5] = 5x^4$$

$$\frac{d}{dx} [x^{-3}] = \frac{d}{dx} [x^{-3}] = -3x^{-4} = \frac{-3}{x^4}$$

$$\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx} [K f(x)] = K f'(x)$$

$$\frac{d}{dx} [5x^3 - 3x] = 5(3x^2) - 3(1x^0)$$
$$= 15x^2 - 3$$

